

## BM-101

### MATHEMATICS

#### GROUP – B

##### (Short Answer Type Questions)

1. If  $\alpha, \beta, \gamma$  be the roots of the equation  $x^3 + 2x^2 + 3x + 4 = 0$ ,  
Then find the equation whose roots are  
 $1 + \frac{1}{\alpha}, 1 + \frac{1}{\beta}$  and  $1 + \frac{1}{\gamma}$ .
2. Evaluate  $\int_0^{\pi/2} x^2 \sin x \, dx$
3. Let  $G = \{ a \in R / -1 < a < 1 \}$ . Define a binary operation  $\otimes$  on  $G$   
by  $a \otimes b = \frac{a+b}{1+ab} \forall a, b \in G$ . Show that  $(G, \otimes)$  is a group.
4. Find the equation of a straight line through the point of intersection of lines  $2x - 3y + 4 = 0$  and  $3x + 4y - 5 = 0$  and that is perpendicular to the line  $6x - 7y + 8 = 0$ .
5. In a survey concerning the smoking habits of consumers it was found that 55% smoke cigarette-A, 50% smoke cigarette-B, 42% smoke cigarette-C, 28% smoke cigarette-A & B, 20% smoke cigarette-A & C, 12% smoke cigarette-B & C and 10% smoke all the three cigarette. What percentage does no smoke?

#### GROUP – C

##### (Long Answer Type Questions)

6. a) Find the value of 'a' and 'b' for which the system of equations
$$\begin{aligned}x + 2y + z &= 1 \\2x + y + 3z &= b \\x + ay + 3z &= b + 1\end{aligned}$$
has (i) unique solution, (ii) many solutions.
- b) If  $P = \begin{pmatrix} 9 & 1 \\ 4 & 3 \end{pmatrix}$  and  $Q = \begin{pmatrix} 1 & 5 \\ 7 & 12 \end{pmatrix}$ , find the matrix  $R$  so that  $5P + 3Q + 2R$  is a null matrix.
- c) Find the inverse of the following matrix

$$\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}$$

7. a) If  $u = \cos^{-1} \left\{ \frac{(x+y)}{\sqrt{x} + \sqrt{y}} \right\}$ , then prove that

$$x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} + \frac{1}{2} \cot u = 0$$

b) If PSQ be a focal chord of a conic with focus S and semi latus rectum L, then prove that

$$\frac{1}{SP} + \frac{2}{SQ} = \frac{2}{L}$$

c) Find the point on the conic  $\frac{6}{r} = 1 + 4 \cos \theta$  whose vertical angle is  $\pi/3$ .

8. a) Evaluate

$$\lim_{n \rightarrow \infty} \left[ \frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \dots + \frac{n}{n^2+n^2} \right].$$

b) Differentiate n times the following equation:

$$(1 + x^2)y_{n+2} + (2x - 1)y_1 = 0$$

c) If  $A = \{a, b, c, d\}$ ,  $B = \{b, c, p, q\}$ , then find out  $A \times B$ ,  $B \times A$  and  $A \Delta B$ .