5. Equity Valuation and the Cost of Capital

Introduction

Part Two provided a detailed explanation of the *investment decision* with only oblique reference to the *finance decision*, which determines a company's cost of capital (discount rate) designed to maximise shareholder wealth. But if wealth is to be maximised, management must determine what return their shareholders require from an investment and then only accept projects that have a positive NPV when discounted at that rate.

There is also the question as to what cut-off rate should apply to investment proposals if corporate finance were obtained from a *variety* of sources, other than ordinary shares? Each stakeholder requires a rate of return that may differ from the equity market and may be unique. In this newly *leveraged* situation, the company's *overall* cost of capital (rather than its cost of equity) measured by its *weighted*, *average cost of capital* (WACC) would seem to be the appropriate investment acceptance criterion.

Given the normative assumption of financial management, the purpose of Part Three is straightforward. How does a firm *maximise* corporate wealth by securing funds at *minimum* cost that not only provides *shareholders* with their desired rate of return, once investment takes place, but also satisfies the expectations of all *capital providers*?

To set the scene, Chapter Five provides an explanation of the most significant *explicit, opportunity cost* of external funding available to management. The cost of ordinary shares measured by their rate of return, often termed the *equity capitalisation rate* or *yield.*

5.1 The Capitalisation Concept

In Chapter Two we defined an investment's present value (PV) as its relevant periodic cash flows (Ct) discounted at a constant cost of capital (r) over time (n). Expressed algebraically:

$$(1) \quad PV_n = \sum_{t=1}^{n} C_t / (1+r)^t$$

The equation has a convenient property. If the investment's annual cash receipts are also *constant and tend to infinity*, ($C_t = C_1 = C_2 = C_3 = C_\infty$) their PV simplifies to the formula for the *capitalisation of a constant perpetual annuity*:

$$(2) \quad PV_\infty = C_t / r$$

The term r is called the *capitalisation* rate because the transformation of a cash flow series to value (i.e. capital) is termed "capitalisation". With data on PV_∞ and r, or PV_∞ and C_t, we can also determine values for Ct or r respectively. Rearranging Equation (2) with one unknown:

$$(3) \quad C_t = PV_\infty \, . \, r$$

$$(4) \quad r = PV_\infty / C_t$$

These PV equations are vital to your understanding of various share valuation models, which define the possible cost of equity as a managerial cut-off rate for investment. So, let us define the models beginning with dividend valuation.

5.2 *Single*-Period Dividend Valuation

Assume you hold a share for *one year*, at the end of which a dividend is paid. You then sell the share *ex-div,* which means the new investor does not receive the dividend (you do) as opposed to *cum-div*, where the dividend is incorporated into price. Your current *ex-div* price, (P_0) is defined by the expected year-end dividend (D_1) *plus* the expected year-end share price (P_1) discounted at the appropriate rate of return for shares in that risk class, the cost of equity (K_e). Thus, we have the *single-period dividend valuation model*:

(5) $P_0 = (D_1 + P_1) / 1 + K_e = [(D_1 / 1 + K_e) + (P_1 / 1 + K_e)]$

Sequentially, if the new investor holds the share for a further year, then their *ex-div* price on acquisition (i.e. dated when you sold it) is also given by the single- period model.

(6) $P_1 = [(D_2 / 1 + K_e) + (P_2 / 1 + K_e)]$

Note however that if you held the share for two years, its current *ex-div* price would be the discounted sum of two dividends and the *ex-div* price at the end of year two, as follows:

(7) $P_0 = [(D_1 / 1 + K_e) + (D_2 / 1 + K_e)^2 + (P_2 / 1 + K_e)^2]$

5.3 *Finite* Dividend Valuation

Assuming the cost of equity K_e is constant; the current *ex-div* price of a share held for any *finite* number of years (n) and then sold equals:

(8) $P_0 = [(D_1 / 1 + K_e) + (D_2 / 1 + K_e)^2 + \ldots.. + (D_n / 1 + K_e)^n] + (P_n / 1 + K_e)^n$

Rewritten, this defines the *finite-period, dividend valuation model:*

(9) $$P_0 = \sum_{t=1}^{n} D_t / (1+K_e)^t + P_n / (1 + K_e)^n$$

where P_n equals the *ex-div* value at time period n, determined by the discounted sum of subsequent dividends.

(10) $P_n = [\{D_{n+1}/(1+K_e)^{n+1}\} + \{D_{n+2} /(1+K_e)^{n+2}\} + \ldots..]$

Activity 1

A potential shareholder anticipates a dividend per share of 10 pence and 11 pence in years one and two respectively, whereupon the shares are expected to be sold *ex div* for £3.00 each. If the equity capitalisation rate is 20 percent per annum, confirm that the maximum current *ex-div* price at which the shares should be purchased is £2.24.

5.4 *General* Dividend Valuation

If distributions tend to infinity, then by definition the final term of Equation (9) disappears altogether because the share is never sold. This is the *general dividend valuation model*:

$$(11)\quad P_0 = \sum_{t=1}^{\infty} D_t / (1+K_e)^t$$

5.5 *Constant* Dividend Valuation

Finally, if we assume that dividends are *constant* in perpetuity ($D_t = D_1 = D_2 = D_3 \ldots = D_\infty$) and K_e is constant then the *general* model simplifies to the *constant dividend valuation model*.

(12) $P_0 = D_1 / K_e$

5.6 The Dividend Yield and Corporate Cost of Equity

We stated earlier that an appreciation of equity valuation models is a pre-requisite for understanding why shareholder returns provide the management of an *all-equity* firm with its cut-off rate for investment. To prove the point, let us rearrange the terms of Equation (12).

(13) $K_e = D_1 / P_0$

We have now defined the *dividend yield* published daily by the financial press throughout the world from stock exchange listings. Whilst the yield is based on an *abstract* constant dividend model, its use by investors as a corporate performance indicator is rational.

In an uncertain world where future dividend or price movements are unknown, it is reasonable to assume that without information to the contrary, future returns should at least equal today's ratio of a company's latest dividend to current share price. As a percentage, this dividend yield also enables investors to compare a company's performance over time, with its competitors, or the market, to establish whether its shares are over or under valued.

> A "golden" investment rule is *the higher the risk, the higher the return and lower the price*. For example, a firm declares a 20 pence dividend on shares currently trading at £2.00. The yield is 10 per cent. But shareholders interpret the dividend as "bad" news and after panic selling, price falls to £1.00. So, the yield doubles, not because of improved performance but increased risk. Investors are now paying *less* for the *same* dividend.

Management ignore dividend yields at their peril

Because dissatisfied shareholders can always seek investment opportunities elsewhere, the percentage dividend for every £100 they invest in a company should represent a managerial *benchmark* for accepting new projects of equivalent risk. The yield also represents a *minimum* project return if management retain profits for reinvestment, rather than pay a dividend. Recalling *Fisher's Separation Theorem* and *Agency Theory*, firms that cannot *maintain* yields should distribute profits for shareholders to reinvest on the capital market. To summarise:

The *current* dividend yield is an *opportunity* cost of capital which equals the *minimum* cut-off (discount) rate for new investment in an all-equity firm.

5.7 Dividend Growth and the Cost of Equity

For a company, the shareholder concept of *maintainable yield* based on the *constant dividend* model provides a convenient discount rate. Unfortunately, it is too simplistic. Assuming $D_1 = D_2 = D_3$ and so on, implies either a 100 percent dividend pay-out ratio or zero-growth, both of which are rarely observed in the real world. Most firms retain a proportion of earnings for profitable reinvestment to enhance shareholder wealth through dividend growth and capital gains. So, how does this affect the yield as a cut-off rate for investment?

Beginning with a valuation model let us assume that through retention financed investment dividends now grow at *a constant annual compound rate (g) in perpetuity*. Leaving aside the detailed mathematics (that you can download elsewhere) M J. Gordon (1958) proved that the current *ex-div* price becomes:

(14) $P_0 = D_1 / K_e - g$ (subject to the non-negativity constraint that $K_e > g$)

The Gordon *constant growth dividend model* defines the current *ex-div* share price by capitalising next year's dividend at the amount by which the desired equity return *exceeds* the constant rate of growth in dividend.

For example, if we assume that the next dividend per share is 20 pence, the shareholders' rate of return is 10 percent per annum and the annual growth rate is five percent

$$P_0 = D_1 / K_e - g = £0.20 / (0.10 - 0.05) = £4.00$$

Activity 2

Take growth out of the previous equation or use Equation (12) for the constant dividend model to confirm that P0 is only £2.00. What does this reveal?

The two equations illustrate an important consideration for rational investors when buying shares, namely how growth potential can uplift equity value.

Rearrange the terms of Equation (14) and we can also isolate the impact of constant growth on the shareholders overall return.

(15) $K_e = (D_1 / P_0) + g$

So now, the cost of equity as a managerial discount rate equals a dividend expectation divided by current share price, *plus a premium for growth*. Using our previous example:

$$K_e = (£0.20 /£4.00) + 0.05 = 5\% + 5\% = 10\%$$

5.8 Capital Growth and the Cost of Equity

Because dividend growth increases price, we can reformulate Equations (14) and (15) by

focussing on the *capital gain* impact on equity value and the corporate cut-off rate rate.

If share price grows at a constant annual rate $G = (P_1 - P_0) / P_0$ then next year's price:
(16) $P_1 = P_0 (1 + G)$

From Equation (14) we also know that the current price based on dividend growth (g):

(14) $P_0 = D_1 / K_e - g$

So, logically share price one year from now must equal:

(17) $P_1 = D_2 / K_e - g = D_1 (1+g) / K_e - g$

Because the same share cannot sell at different prices, it follows from Equation (16) that the dividend growth rate (g) must equal (G) the annual growth in share price (capital gain). Equations (16) and (17) can therefore be redefined as follows:

(18) $P_1 = P_0 (1+g)$

A comparison of Equations (16) and (18) reveals that if share price grows at a rate G, this must equal g, the annual growth in dividends. If we substitute G for g into Equation (14), this produces a *dividend-capital gain model* equivalent to the Gordon growth model.

(19) $P_0 = D_1 / K_e - G$

The current *ex div* share price is determined by capitalising next year's dividend at the amount by which the desired rate of return on equity exceeds the percentage *capital gain*.

Activity 3

If a company's forecast dividend is 20 pence per share, price is expected to grow at five percent per annum, and the equity capitalisation rate is 10 per cent:

P0 = D1 / Ke -G = £0.20 / (0.10 - 0.05) = £4.00

Use the Gordon growth model to confirm that the current ex-div price still equals £4.00

Turning now to an equity capitalisation rate, which incorporates capital gains as a managerial investment criterion, we can substitute G for g into Equation (14) and rearrange terms so that:

(20) $K_e = (D_1 / P_0) + G$ [from $P_0 = D_1 / K_e - g$ and $K_e = (D_1 / P_0) + g$]

This equation states that the *total* cost of equity comprises a *dividend yield* one year hence (D_1/P_0), plus a *capital gain yield* [$G = (P_1 - P_0) / P_0$] equivalent to the growth in dividends (g).

Activity 4

If a company currently trading at £4.00 per share with a forecast 20 pence dividend is expected to grow at five percent per annum, confirm that the equity capitalisation rate is 10 per cent using the appropriate *dividend and capital gain* models.

5.9 Growth Estimates and the Cut-Off Rate

So far so good, but if management wish to finance future projects by retaining profits in their quest for shareholder wealth, how do they calculate the growth rate?

Obviously, dividend and capital gains are rarely constant, which gives rise to complex valuation models that are beyond the scope of this text. But even if they are uniform, management still need annual growth estimators. Since the future is so uncertain, a simple solution favoured by management is to assume that the past and future are *interdependent*. Without information to the contrary, Gordon (*op cit*) believed that a company's anticipated growth should be determined from its financial history. Consider the following data:

Year	**Dividend per Share (pence)**
2005	20
2006	22
2007	24.2
2008	26.62
2009	29.28

Using the formula $(D_t\text{-}D_{t-1})/D_{t-1}$ we can determine annual dividend growth rates

Year	**Annual Growth Rate**	
2005-6	(22/20) -1	= 0.1
2006-7	(24.2/22) -1	= 0.1
2007-8	(26.62/24.2) -1	= 0.1
2008-9	(29.28/26.62)-1	= 0.1
Total		0.4

The *average* periodic growth rate, as an estimator of g, is therefore given by:

$g = 0.4 / 4 = 10\%$

Alternatively, we can calculate dividend growth by solving for g in the following equation and rearranging terms.

$20 \text{ pence } (1+g)^4 = 29.28 \text{ pence.}$

$:(1 + g) = \sqrt[4]{(29.28/20.00)}$

$g = 1.10 - 1.00 = 0.10 = 10\%$

Activity 5

Using the previous data and the appropriate equations, confirm that the forecast dividend for 2010 should be 32.21 pence. If shares are currently priced at £2.68 and dividends are expected to grow at ten percent per annum beyond 2010, confirm that the equity capitalisation rate (managerial cut-off rate for new investment) is 22 per cent.

5.10 Earnings Valuation and the Cut-Off Rate

Whether or not growth is incorporated into the model, there is still no consensus as to whether dividends alone determine a share's value and hence the firm's cut-off rate for investment.

As long ago as 1961, the Nobel economic prize winners, Franco Modigliani and Merton Miller (MM) argued that given the problems of estimating retention-financed dividend growth, why not assume that dividends and retentions are *perfect economic substitutes*? Because if so; a company's share price and capitalisation rate can be determined by its *overall earnings*, rather than dividend policy. Since the future is uncertain, they also recommended a *one period model*.

> *According to MM*, the current *ex div* share price (P_0) equals the anticipated earnings per share (E_1) plus the *ex div* price (P_1) at the end of the year, discounted at the shareholders' rate of return (K_e).Algebraically, their *single-period earnings model* is:
>
> (21) $P_0 = (E_1 + P_1) / 1 + K_e = [(E_1 / 1 + K_e) + (P_1 / 1 + K_e)]$

Of course, earnings (like dividend) proponents confident with their forecasts need not restrict themselves to one period, or zero- growth. Assuming the cost of equity K_e is constant, the current *ex-div* price of a share held for any *finite* number of years (n) and then sold ex-div for P_n equals the *finite-period earnings model*

(22) $$P_0 = \sum_{t=1}^{n} E_t / (1+K_e)^t + P_n / (1 + K_e)^n$$

If n tends to infinity, then the *general earnings valuation model* is given by

(23) $$P_0 = \sum_{t=1}^{n} E_t / (1+K_e)^t$$

If annual earnings E_t are constant in perpetuity, Equation (23) simplifies to the *constant earnings valuation model*:

(24) $P_0 = E_1 / K_e$

We can also incorporate growth into the previous equation to derive a *constant earnings growth model* analogous to the *Gordon dividend model* such that:

(25) $P_0 = E_1 / K_e - g$ (again subject to the non-negativity constraint that $K_e > g$)

Review Activity

The only apparent difference between Equations (21) to (25) and our earlier dividend valuation models is the substitution of an earnings term (E) for dividends (D) in a *parallel* series of equations. However, because the *same share cannot trade at two prices*, the reformulation of corresponding P_0 equations to derive the cost of equity (K_e) may have important consequences for the managerial cut-off rate. Can you explain why?

If a company adopts a *full* distribution policy, where dividend per share *equals* earnings per share, then substituting E_t for D_t into either valuation models has no effect on the cost of equity (K_e).For example, reformulating the *constant valuation model* that solves for P_0:

If $D_t = E_t$ then $P_0 = D_t / K_e = P_0 = E_t / K_e$ and $K_e = D_t / P_0 = K_e = E_t / P_0$

But what if a company adopts a *partial* distribution policy (where $D_t < E_t$,).

Because the *same* share cannot trade at two prices, the equity return (K_e) must *differ* in the corresponding dividend and earnings equations if P_0 is to remain the same. Mathematically:

If $D_t < E_t$ but $P_0 = D_t / K_e = P_0 = E_t / K_e$ then $K_e = D_t / P_0 < E_t / P_0$

Moreover, if P_0 is identical throughout both series of dividend and earnings value equations, outlined earlier, then not only must the equity yield for dividends and earnings (K_e) differ, but a *unique* relationship must also exist between the two.

For example, if a *dividend yield* equals 10 percent per annum in response to a dividend of £1.00, the current share price should be

$P_0 = D_t / K_e = £1.00 / 0.1 = £10.00$

But if we now assume the *dividend-payout* ratio is 50 per cent and substitute the annual earnings per share of £2.00 into the previous equation, then subject to the *law of one price* (where P_0 still equals £10.00) we produce the following equation with *one unknown*.

$P_0 = E_t / K_e = £2.00 / K_e = £10.00$

Rearranging terms, we can therefore define the *earnings yield* as an alternative to dividends as a managerial cut-off (discount) rate for new investment.

(26) $K_e = E_t / P_0$

And solving for the earnings yield, we observe a difference to the dividend yield

$$K_e = E_t / P_0 = £2.00 / £10.00 = 20\% \quad > \quad Ke = D_t / K_e = £1.00 / £10.00 = 10\%$$

Not only do the two yields differ but note they exhibit an *inverse* relationship defined by the dividend payout (earnings retention) ratio. Because the same share cannot sell at different prices and the dividend per share is *half* the earnings per share, then the earnings yield must be *twice* the dividend yield.

5.11 Summary and Conclusions

We began our study of strategic financial management way back in Part One with an explanation of how companies employ their overall cost of capital as an investment criterion designed to maximise shareholder wealth. You will recall that under conditions of reasonably perfect markets, certainty and equilibrium, the correct cost is defined as the minimum return required by investors from an alternative investment of equivalent risk (The Separation Theorem of Fisher).So, if an *all equity* company undertakes a capital project using the marginal cost of equity as its discount rate, the total market value of ordinary shares should increase by the project's NPV.

In this Chapter we therefore addressed the crucial issue of equity valuation and the derivation of its associated capital cost as a discount rate, from both a dividend and earnings perspective under growth and non-growth conditions. We concluded that an equity capitalisation rate based on earnings, rather than dividends, should be management's preferred cut-off rate for new investment. But what if fund sources other than share capital are available to management. How do these affect project discount rates in our newly leveraged firm?