

Name :

Roll No. :

Invigilator's Signature :

CS/BCA/SEM-1/BM-101/2010-11
2010-11
MATHEMATICS

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

GROUP - A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : 10 × 1 = 10

i) $\lim_{x \rightarrow 0} (1+x)^{1/x} = ?$

a) 1

b) 0

c) $\frac{2}{3}$

d) e.

ii) If α, β, γ be the roots of the equation $x^3 + yx + z = 2$ then $\Sigma x^2 =$

a) 0

b) 14

c) -14

d) 4.

- iii) An element x in a ring R is zero divisor if
- a) $x \cdot b = 0$
 - b) $x \cdot b = 0$, for some non zero element b in R
 - c) $x \cdot b \neq 0$, for all element b in R
 - d) none of these.
- iv) The value of $\int_{-1}^2 |x| dx$ is
- a) 3
 - b) 5
 - c) $\frac{5}{2}$
 - d) 0.
- v) The value of $\frac{d}{dx}(\log_e x)$ is equals to
- a) $\frac{1}{x}$
 - b) $\log\left(\frac{1}{x}\right)$
 - c) $\left(\frac{1}{n}\right) \log_a e$
 - d) $a \log e$.
- vi) If $A = \{2, 4, 6\}$ and $B = \{1, 3, 5, 7\}$, then $A \cup B$ is
- a) $\{0\}$
 - b) $\{1, 2, 3, 4, 5, 6, 7\}$
 - c) $\{1, 2, 4, 5, 6, 7\}$
 - d) $\{0, 2\}$.
- vii) If A is a square matrix then
- a) $A + A^T$ is symmetric
 - b) $A + A^T$ is skew symmetric
 - c) $A - A^T$ is symmetric
 - d) $A - A^T$ is skew symmetric.

- viii) The matrix $A = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$ is on
- a) orthogonal matrix b) idempotent matrix
 c) identity matrix d) none of these.
- ix) If $y = 2$ at and $x = at^2$, then $\frac{dy}{dx}$ at $t = 1$ is
- a) 1 b) $2a$
 c) -1 d) $2a^2$.
- x) The polar form of the equation $x^2 + y^2 - 8y = 0$ is
- a) $r = 8 \cos \theta$ b) $r = 8 \sin \theta$
 c) $r^2 = 8 \cos \theta$ d) none of these.
- xi) If $A = \{1, 2, 3, 4, 8\}$, $B = \{2, 4, 6, 7\}$ then $A \Delta B$ is
- a) $\{2, 4\}$
 b) $\{1, 2, 3, 4, 6, 7, 8\}$
 c) ϕ
 d) $\{1, 3, 6, 7, 8\}$.
- xii) The diagonal elements of a real skew-symmetric matrix are
- a) 1 b) -1
 c) 2 d) 0.

GROUP - B

(Short Answer Type Questions)

Answer any *three* of the following.

3 × 5 = 15

2. A function $f(x)$ is defined as follows

$$f(x) = x^2 \quad \text{when } 0 < x < 1$$

$$= x \quad \text{when } 1 \leq x < 2$$

$$= 2 - x \quad \text{when } 2 \leq x < 3$$

Show that the $f(x)$ is continuous at $x = 2$.

3. Evaluate $\int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$.

4. If α, β, γ be the roots of the cubic $x^3 + px + q = 0$, then find the equation whose roots are

$$\frac{\beta + \gamma}{\alpha^2}, \frac{\gamma + \alpha}{\beta^2}, \frac{\alpha + \beta}{\gamma^2}.$$

5. Prove that the ring of matrices of the form $\begin{bmatrix} x & y \\ -y & x \end{bmatrix}$ of real number is a field.

6. In a survey concerning the smoking habits of consumers it was found that 55% smoke cigarette-A, 50% smoke cigarette-B, 42% smoke cigarette-C, 28% smoke cigarette-A & B, 20% smoke cigarette-A & C, 12% smoke cigarette-B & C and 10% smoke all the three cigarette. What percentage do not smoke ?

GROUP - C**(Long Answer Type Questions)**Answer any *three* of the following. $3 \times 15 = 45$

7. a) If $y = \sin (m \sin^{-1} x)$, then show that
 $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$.
- b) If α, β, γ are the 3 roots of $x^3 + px^2 + qx + r = 0$ obtain the value of $\sum(\alpha - \beta)^2$.
- c) Evaluate $\int \frac{1}{x^2} e^{1/x} dx$.
8. a) If $u = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$ then prove that, $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$.
- b) If by a rotation of rectangular co-ordinate axes without change of origin expressions $ax + by$ and $cx + dy$ are transformed into $a'x' + b'y'$ and $c'x' + d'y'$. Show that $a'd' - b'c' = ad - bc$.
- c) Reduce the following equation to its canonical form and determine the nature of the conic represented by it :
 $3x^2 - 8xy - 3y^2 + 10x - 13y + 18 = 0$

9. a) Evaluate

$$\lim_{n \rightarrow \infty} \left[\frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \dots + \frac{n}{n^2+n^2} \right]$$

b) Using mean value theorem prove the following

inequality :

$$x \left(\sin^{-1} x \right) < \frac{x}{\sqrt{1-x^2}}, \text{ if } 0 < x < 1$$

c) Expand $\sin x$ in power of x in infinite series.

10. a) Solve the equation by Cardan's method :

$$2x^3 + 3x^2 + 3x + 1$$

b) Evaluate

$$\int \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)}$$

c) If $y = x^{x-1} \log x$, show that $y_x = \frac{(x-1)!}{x}$.

11. a) Prove that $|A \cup B| = |A| + |B| - |A \cap B|$ where A and B are two non-empty sets.
- b) If $A = \{a, b, c, d\}$ $B = \{b, c, p, q\}$, then find out $A \times B$, $B \times A$ and $A \Delta B$.
- c) Define power set. Find the power set of $\{a, b, c\}$.
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