Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/BCA/SEM-1/BM-101/2009-10

2009

MATHEMATICS

Time Allotted : 3 Hours — Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) The value of $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$ is

a) 1 b) 4

c) 0 d) 2.

ii) The value of $\int_1^2 dx$ is equal to

a) 1 b) 2

c) 3 d) 0.

iii) If $x = -1$ is a root of the equation $x^2 - x - k = 0$, then the value of k is

a) 1 b) 0

c) $\sqrt{2}$ d) 2.

iv) If α, β, γ be the roots of the equation

$x^3 - 3x^2 + 6x - 2 = 0$, then $\alpha + \beta + \gamma$ is

a) 2 b) 1

c) 3 d) none of these.

v) If $A\{1, 2, 3, 4\}$ and $B = \{2, 4, 6\}$, then $A \Delta B$ is

a) $\{1, 2\}$ b) $\{1, 2, 3, 6\}$

c) $\{1, 3, 6\}$ d) $\{6\}$.

vi) What is the order of the matrix B, if $[3\ \ 4\ \ 2]$

$B = [2\ \ 10\ \ 3\ \ 6\ \ 9]$?

a) 1×5 b) 1×3

c) 3×5 d) 5×3.

vii) The degree of the polynomial

$f(x) = (x^2 + x - 2) / (x - 1)$ is

a) 0 b) 1

c) 2 d) 3.

viii) If $y = \log x^2$, the value of $\frac{d^2y}{dx^2}$ is

a) $\frac{2}{x^3}$ b) $-\frac{2}{x^2}$

c) $\frac{2}{x}$ d) $2x$.

ix) The value of t for which the matrix $\begin{bmatrix} 2 & 0 & 1 \\ 5 & t & 3 \\ 0 & 3 & 1 \end{bmatrix}$ is singular, is

a) $-\frac{3}{2}$ b) 2

c) $\frac{3}{2}$ d) -2.

x) $\lim_{x \to 0} (1 + x)^{1/x}$ is equal to

a) 1 b) e

c) ∞ d) 0.

xi) If α, β, γ be the roots of the equation

$x^3 - 3x^2 + 6x - 2 = 0$, then $\Sigma \alpha\beta$ is

a) 3 b) 6

c) 2 d) none of these.

xii) If $A \{1, 2, 3\}$ and $B = \{2, 3, 6\}$, then $A \cup B$ is

a) $\{1, 2, 3\}$ b) $\{2, 3\}$

c) $\{1, 2, 3, 6\}$ d) none of these.

GROUP – B
(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Evaluate the integral $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} \, dx$.

3. If $u = \log r$ and $r^2 = x^2 + y^2 + z^2$, prove that

$$r^2 \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right] = 1.$$

4. In a survey of 320 persons, number of persons taking tea is 210, taking milk is 100 and coffee is 70. Number of persons who take tea and milk is 50, milk and coffee is 30, tea and coffee is 50. The number of persons taking all three together is 20. Find the number of people who take neither tea nor coffee nor milk.

5. Express $\begin{bmatrix} -3 & 4 & 1 \\ 2 & 3 & 0 \\ 1 & 4 & 5 \end{bmatrix}$ as a sum of a symmetric and a skew-symmetric matrix.

6. If α, β, γ be the roots of the equation $x^3 + 2x^2 + 3x + 4 = 0$, then find the equation whose roots are

$$1 + \frac{1}{\alpha}, \quad 1 + \frac{1}{\beta} \quad \text{and} \quad 1 + \frac{1}{\gamma}.$$

GROUP – C
(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Verify whether the matrix $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

b) Solve the following system of linear equations by using Cramer's Rule :

$$2x + 5y + 3z = 9$$

$$3x + y + 2z = 3$$

$$x + 2y - z = 6$$

c) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 2 & -2 \\ 3 & 3 \end{bmatrix}$, find AB.

d) Show that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \sin\theta \end{bmatrix} \times \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \sin\theta \end{bmatrix}$

$$= \begin{bmatrix} \cos 2\theta & -\sin\theta(\sin\theta + \cos\theta) \\ \sin\theta(\sin\theta + \cos\theta) & 0 \end{bmatrix}$$

2 + 5 + 4 + 4

8. a) Evaluate any *two* :

i) $\lim_{x \to 0} \dfrac{\tan 2x - x}{3x - \sin x}$

ii) $\lim_{x \to 0} \dfrac{x \log \sqrt{1 + x}}{\sin^2 x}$

iii) $\lim_{x \to a} \dfrac{1 - \cos(x - a)}{(x - a)^2}$

b) Evaluate $\displaystyle\int_0^{\pi/2} x^2 \sin x \, dx$.

c) Differentiate $\dfrac{x^3}{(1 + x^3)}$ with respect to x^4.

5 + 5 + 5

9. a) If $A = \{ a, b, c, d, e \}$, $B = \{ c, a, e, g \}$ and $C = \{ b, e, f, g \}$, then show that

$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C).$$

b) If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, then find A^2 and show that $A^2 = A^{-1}$.

c) Find the maxima and minima of $x^3 - 6x^2 + 9x - 8$.

5 + 5 + 5

10. a) Determine whether the function

$$f(x, y) = \frac{xy}{x^2 + y^2} \quad \text{if } (x, y) \neq (0, 0)$$

$$= 0 \quad \text{if } (x, y) = (0, 0)$$

is continuous at the origin.

b) Apply Descartes' rule of signs to find the nature of roots of the equation

$$x^4 + 2x^2 + x - 12 = 0$$

c) State Cauchy's mean value theorem. 5 + 5 + 5

11. a) Find the value of 'a' and 'b' for which the system of equations

$$x + 2y + z = 1$$

$$2x + y + 3z = b$$

$$x + ay + 3z = b + 1$$

has (i) unique solution, (ii) many solutions.

b) Solve the following system of equations by matrix inversion method

$$x + y + z = 6$$

$$x - 2y + z = 0$$

$$2x - y + z = 3$$

c) Find out the rank of the matrix $\begin{bmatrix} 2 & -4 & 6 \\ 2 & 3 & -1 \\ 3 & 1 & 2 \end{bmatrix}$.

5 + 5 + 5

12. a) If $u = \cos^{-1}\left\{ (x+y) / \sqrt{x} + \sqrt{y} \right\}$, then show that

$$x.\frac{\partial u}{\partial x} + y.\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0$$

b) If PSQ be a focal chord of a conic with focus S and semi latus rectum L, then prove that

$$1/SP + 2/SQ = 2/L.$$

c) Find the point on the conic $6/r = 1 + 4\cos\theta$ whose vertical angle is $\pi/3$.

8 + 4 + 3