ENGINEERING & MANAGEMENT EXAMINATIONS, DECEMBER – 2008

MATHEMATICS – I

SEMESTER – 1

Time : 3 Hours] [Full Marks : 70

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) If $A = \{ 1, 2, 3, 4, 8 \}$, $B = \{ 2, 4, 6, 7 \}$, then $A \Delta B$ is

a) $\{ 2, 4 \}$

b) $\{ 1, 2, 3, 4, 6, 7, 8 \}$

c) ϕ

d) $\{ 1, 3, 6, 7, 8 \}$.

ii) $\lim_{x \to 0} (1 + x)^{1/x}$ is equal to

a) 1 b) e

c) 0 d) ∞ .

iii) $\frac{d}{dx} \left(\log_a x \right)$ is equal to

a) $\frac{1}{x}$ b) $\log (1/x)$

c) $(1/x) \log_a e$ d) $x \log e$.

iv) If $y = \log x^2$, the value of $\frac{d^2 y}{dx^2}$ is

a) $\frac{2}{x^2}$ b) $-\frac{2}{x^2}$

c) $\frac{2}{x}$ d) $2x$.

v) The matrix $A = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ is an

a) orthogonal matrix
b) idempotent matrix
c) identity matrix
d) none of these.

vi) Derivative of x^4 with respect to x^2 is

a) $4x^3$
b) $2x^2$
c) $2x$
d) 4.

vii) If the roots of the equation $ax^2 + bx + c = 0$ ($a \neq 0$) are real and unequal, then its discriminant D satisfies

a) $D > 0$ and D = a perfect square
b) $D = 0$
c) $D > 0$ and $D \neq$ a perfect square
d) $D < 0$.

viii) If $A = \{ 1, 2, 3 \}$, $B = \{ 2, 3, 6 \}$, then $A \cup B$ is

a) $\{ 1, 2, 3 \}$
b) $\{ 2, 3 \}$
c) $\{ 1, 2, 3, 6 \}$
d) none of these.

ix) If α, β, γ be the roots of $x^3 - 3x^2 + 6x - 2 = 0$, then $\sum \alpha\beta$ is

a) 3
b) 6
c) 2
d) none of these.

x) If $f(x) = 3 + 2x$; when $x \geq 0$

$= -3 - 2x$; when $x < 0$,

then $\lim_{x \to 0} f(x)$ is

a) 3
b) -3
c) 0
d) none of these.

xi) If $f(x) = \frac{x}{|x|}$; when $x \neq 0$

$= 1$; when $x = 0$, then

a) $f(x)$ is continuous at $x = 0$

b) $f(x)$ is continuous, but not differentiable at $x = 0$

c) $f(x)$ is discontinuous at $x = 0$

d) none of these.

xii) The value of $\int_{-1}^{2} |x| dx$ is

a) 3

b) 5

c) 5/2

d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. If α, β, γ be the roots of the equation, $x^3 + px^2 + qx + r = 0$, then find the value of $\Sigma \alpha^3$.

3. If $u = \tan^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{4}\sin 2u$.

4. Prove that the set of even integers (including zero) forms an additive group.

5. Evaluate $\int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$.

6. If $P = \begin{pmatrix} 9 & 1 \\ 4 & 3 \end{pmatrix}$ and $Q = \begin{pmatrix} 1 & 5 \\ 7 & 12 \end{pmatrix}$, find the matrix R so that $5P + 3Q + 2R$ is a null matrix.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following questions. $3 \times 15 = 45$

7\. a) State Rolle's Theorem.

b) Differentiate n times the following equation :

$$\left(1 + x^2\right) y_2 + (2x - 1) y_1 = 0.$$

c) If $y = \sin\left(m \sin^{-1} x\right)$, show that

$$\left(1 - x^2\right) y_{n+2} - (2n + 1) x y_{n+1} + \left(m^2 - n^2\right) y_n = 0.$$

4 + 5 + 6

8\. a) If pth, qth and rth terms of an A.P. are P, Q and R respectively, show that $p(Q - R) + q(R - P) + r(P - Q) = 0$.

b) Show that the centroid of a triangle with vertices $\left(x_1, y_1\right)$, $\left(x_2, y_2\right)$ and $\left(x_3, y_3\right)$ is $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$.

c) Find the equation of a straight line through the point of intersection of lines $2x - 3y + 4 = 0$ and $3x + 4y - 5 = 0$ and that is perpendicular to the line $6x - 7y + 8 = 0$. 5 + 5 + 5

9\. a) Show that $\cos x > 1 - \frac{x^2}{2}$ if $0 < x < \frac{\pi}{2}$.

b) If $f(x, y) = \begin{cases} \frac{x^2 y^2}{x^2 + y^2}, & x^2 + y^2 \neq 0 \\ 0, & x^2 + y^2 = 0, \end{cases}$

then show that $f_{xy}(0, 0) = f_{yx}(0, 0)$.

c) Evaluate $\int \frac{x^2}{\left(x^2 + a^2\right)\left(x^2 + b^2\right)} dx$. 4 + 6 + 5

10\. a) Reduce the following equation to its canonical form and determine the nature of the conic represented by it :

$$6x^2 - 5xy - 6y^2 + 14x + 5y + 4 = 0.$$

b) Find the equation of the ellipse one of whose foci is (– 1, 1), eccentricity is 0·5 and the corresponding directrix is $y = x + 3$. 9 + 6

11. a) Solve the equation by Cardan's method, $2x^3 + 3x^2 + 3x + 1 = 0$.

b) Let $G = \{ a \in R / -1 < a < 1 \}$. Define a binary operation $\otimes$ on G by $a \otimes b = \frac{a+b}{1+ab}$ $\forall$ $a, b \in G$. Show that $(G, \otimes)$ is a group.

c) Find the nature of the roots $x^4 + qx^2 + rx - s = 0$ by Descartes' rule of signs (where q, r, s, being positive). 15

12. a) If by a trnasformation of one rectangular axis to another with same origin the expression $ax + by$ changes to $a'x' + b'y'$,

prove that $a^2 + b^2 = a'^2 + b'^2$,

b) Show that $\int_0^\infty \frac{dx}{(x+1)(x+2)} = \log 2$.

c) Use the method of integration to evaluate $\underset{n \to \infty}{Lt} \frac{1^k + 2^k + \dots + n^k}{n^{k+1}}$; $k > 0$.

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END